OCR
Oxford Cambridge and RSA

GCE

Further Mathematics A

Y534/01: Discrete Mathematics

AS Level

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for *g* should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

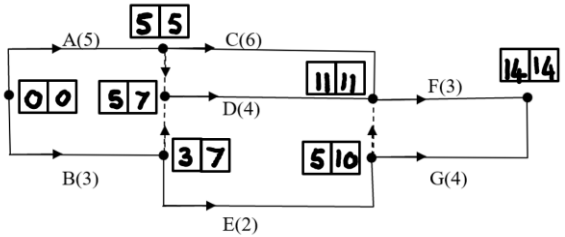
g Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

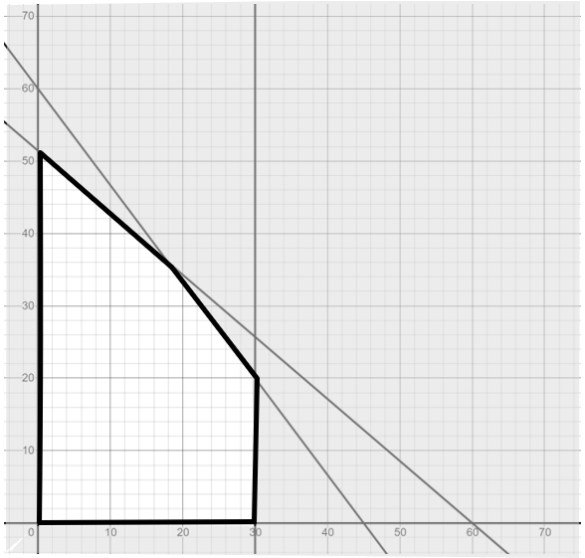
i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

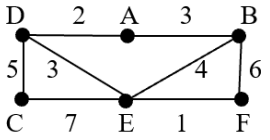
j If in any case the scheme operates with considerable unfairness consult your Team Leader.

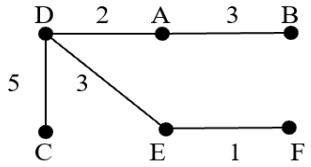
Question			Answer	Marks	AO	Guidance
1	(a)		<table><tr><th>X</th><th>Y</th><th>N</th><th>M</th><th>H</th><th>D</th></tr><tr><td>1</td><td>2</td><td>0</td><td>2</td><td>0.5</td><td>1</td></tr><tr><td>1.5</td><td>2.5</td><td>1</td><td></td><td></td><td>1.13</td></tr><tr><td>2</td><td>3.07</td><td>2</td><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td></tr></table> Display: 3.07	**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	Initial values of *X*, *Y*, *N*, *M*, *H* and *D* First updated values of *X*, *Y*, *D* (with *D* to 3 s.f. or better) (17/15) or seen in table as second updated value of *Y* (to 3 s.f. or better) (46/15)
1	(b)		e.g. *N* is initially 0 and increases by 1 in each pass until it reaches *M*, when the algorithm displays the output and stops	**B1** **[1]**	**2.4**	Explain why algorithm is finite e.g. *N* (starts at 0 and) increases (through positive integer values) until it passes *M*
2	(a)			**B1** **M1** **A1** **[3]**	**1.1** **1.1** **1.1**	Forward pass all correct Backward pass with values not increasing working from end to start cao
2	(b)		<table><tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td></tr><tr><td>0</td><td>4</td><td>0</td><td>2</td><td>5</td><td>0</td><td>5</td></tr></table>	**B1** **B1 FT** **[2]**	**1.1** **1.2**	A, C and F = 0 B, D, E and G correct or ft their values from (a)
2	(c)		Longest route is now BDF = $t + 7$ Maximum value of $t = 9$	**M1** **A1** **[2]**	**3.4** **1.1**	Consider any route that start with B, or (their)7 + (16 – (their) 14) or 16 – (3 + 4), or implied from answer $t = 9$ 9 as answer

Question			Answer	Marks	AO	Guidance
3	(a)	(i)	52 38 15 61 27 49 10 33 96 74 First **38 15 52 27 49 10 33 61 74 96** Second **15 38 27 49 10 33 52 61 74** 96 Third **15 27 38 10 33 49 52 61** 74 96	**M1** **A1** **[2]**	**1.1** **1.1**	First pass of bubble sort correct, condone starting from RHS and/or decreasing, ignore any extra working Correct list at end of second and third passes (increasing starting from LHS) may imply fixed values
3	(a)	(ii)	Comparisons / Swaps First pass: 9 / 7 Second pass: 8 / 5 Third pass: 7 / 3	**B1** **B1** **[2]**	**1.1** **1.1**	Comparisons correct Swaps correct
3	(b)	(i)	52 38 15 61 27 49 10 33 96 74 First **38 52** 15 61 27 49 10 33 96 74 Second **15 38 52** 61 27 49 10 33 96 74 Third **15 38 52 61** 27 49 10 33 96 74	**M1** **A1** **[2]**	**1.1** **1.1**	First pass of shuttle sort correct, condone starting from RHS and/or decreasing, may imply fixed values, ignore extra working Correct list at end of second and third passes, may imply fixed values
3	(b)	(ii)	Comparisons / Swaps First pass: 1 / 1 Second pass: 2 / 2 Third pass: 1 / 0	**B1** **B1** **[2]**	**1.1** **1.1**	Comparisons correct Swaps correct
3	(c)		Fewer comparisons and fewer swaps for shuttle Shuttle sort is more efficient than bubble sort for the first three passes of this list	**M1 FT** **A1 FT** **[2]**	**2.2a** **2.2a**	Compare comparisons and compare swaps (in words or numerically) Shuttle more efficient, or bubble less efficient (at this stage) Follow through their values from (a)(ii) and (b)(ii)

Question			Answer	Marks	AO	Guidance
4	(a)		Sam Kareem 1: 2, 0, -2 2: 1, 4, 4 5: 10, 0, -2	**B1** **M1** **A1** **[3]**	**1.1** **2.1** **2.2a**	May also see working for later parts Correct entries in cells where total is even (1, 3) = 2, (2, 4) and (2, 6) = 4, (5, 3) = 10 (1, 4) = (5, 4) and (1, 6) = (5, 6) Correct entries in cells where total is odd
4	(b)		If S chooses (card) 3, K should play (card) 5 If S chooses (card) 4 or 6, K should play (card) 2	**M1 FT** **A1 FT** **[2]**	**1.1** **1.1**	Any one correct, follow through their pay-off matrix All correct, follow through their pay-off matrix
4	(c)		Row maximin = max{-2, 1, -2} = 1 Kareems's play-safe is the card numbered 2	**M1 FT** **A1 FT** **[2]**	**1.1** **1.1**	Row minima seen, for their pay-offs (may be seen in part (a)) Card 2, from correct working seen, for their pay-offs Need 'card' or equivalent (e.g. strategy)
4	(d)		(Card 1 is) (weakly) dominated by (card) 5	**B1 FT** **[1]**	**2.5**	Or explained in words or showing appropriate comparisons (all) i.e. (Card) 5 because [valid comparisons or explanation] Allow "(Card 1 is) never chosen in part (b)"
4	(e)		K plays 2: $\frac{1}{3}(1 + 4 + 4) = 3$ K plays 5: $\frac{1}{3}(10 + 0 – 2) = \frac{8}{3}$ or $2\frac{2}{3}$	**M1 FT** **A1 FT** **[2]**	**1.1** **1.1**	Either expected score (3 or $2\frac{2}{3}$) for their pay-offs Both expected scores, for their pay-offs

Question			Answer	Marks	AO	Guidance
5	(a)		Maximise $x + y + z$ Subject to	B1	1.1	Max $x + y + z$ Constraints must be given algebraically but may be in any form
			$6x + 7y + 8z \leq 360$	B1	3.3	Jam constraint correct
			$4x + 3y + 2z \leq 180$	B1	1.1	Custard constraint correct
			$x \leq 30$	B1	1.1	At most 30 batches of type X
			$y \geq 2z$	B1	3.1b	At least twice as many batches of type Y as batches of type Z
			$x, y, z \geq 0$			Non-negativity may be implied Use of strict inequalities (e.g. $x < 30$) penalised once only
				[5]		
5	(b)		Set $z = 0$ and graph constraints $6x + 7y \leq 360$, $4x + 3y \leq 180$, $x \leq 30$ and $x, y \geq 0$			
			 	M1 FT	3.5c	At least two of their non-trivial boundary lines correct (ft from above)
				A1	1.1	Feasible region correct (0, 51.4) to (18, 36) to (30, 20) to (30, 0) May assume boundary lines on axes. May see extra lines for part (d)
				M1 FT	3.4	Solving for optimum values of x and y from their graph (checking vertices or profit line or implied from answer)
			18 batches of type X and 36 batches of type Y	A1	3.4	Correct and in context (… batches of type …) Need not state 0 batches of type Z
				[4]		

Question			Answer	Marks	AO	Guidance
5	(c)		e.g. The baker may not sell them all and then they would be wasted e.g. There may be customers who want type Z e.g. The baker may not have enough time	B1 [1]	3.5b	A valid and relevant practical problem
5	(d)		$6x + 7y = k$ passes through (30, 20) when $k = 320$ $6x + 7y = k$ passes through (30, 0) when $k = 180$ $180 \leq k < 320$	 M1 M1 A1 [3]	 2.4 2.4 1.1	Working may be on graph for part (b) 320 as critical value for $4x + 3y \leq 180$ to become redundant 180 as critical value for $x \leq 30$ to become redundant $180 \leq k < 320$ or '$180 \leq k$ and $k < 320$'
6	(a)		B – A – D – C	B1 [1]	2.2b	This route written
6	(b)		Route: A – D – E – F A: 1 | (0) — 1 B: 3 | 3 — 3 C: 7 D: 2 | 2 — 2 E: 4 | 5 — 5 F: 5 | 6 — 9 6	B1 M1 A1	1.1 1.1 1.1	Correct route  Need not be drawn Working on a network or in a table or list Use Dijkstra starting at A All permanent values correct (may also see C = 7)
			Alternative method Route: A – D – E - F A: 5 | 6 — 6 B: 4 | 5 — 6 5 C: 8 D: 3 | 4 — 4 E: 2 | 1 — 1 F: 1 | (0)	B1 M1 A1		Correct route Use Dijkstra starting at F All Permanent values correct (may also see C = 8)
				[3]		

Question			Answer	Marks	AO	Guidance
6	(c)		AB, AD, CD, DE, EF  	**M1** **A1**	**3.1a** **1.1**	A list of five valid arcs that form a tree connecting the six vertices or a tree drawn connecting the six vertices Arcs in MST correct (written or drawn)
			Shortest routes between pairs of villages (when not a single arc) ADC ADE ADEF BADC BAD BEF CEF DEF (or reversed)	**B1**	**3.1a**	At least two of these routes (when not a single arc) May be evidenced by sight of network or graph or any correct pair in answer
			Pairs of villages: B and E, B and F, C and E C and F	**M1** **A1** **[5]**	**3.2a** **3.2a**	Any two correct pairs written in answer (and at most 6 pairs) All four correct <u>and no others</u> (allow BE, BF, CE, CF) Ignore repeats (e.g. B and E, E and B)
7	(a)		{ABCDE} {ABCE | D} {ABDE | C} {ACDE | B} {ABE | CD} {ACE | BD} {ADE | BC} {ABE | C | D} {ACE | B | D} {ADE | B | C} {AE | BCD} {AE | BC | D} {AE | BD | C} {AE | B | CD} {AE | B | C | D}	**M1** **A1** **A1** **[3]**	**1.1** **1.1** **1.1**	At least three correct partitions, in any unambiguous form At least three correct partitions with different structures All 15 correct with no extras or duplicates

Question			Answer	Marks	AO	Guidance
7	(b)		Partition the E in {ABCDE} to get {ABCD | E} For each of the 7 partitions from part (a) into 2 subsets put E in the other subset e.g. {ABCE | D} becomes {ABC | DE}	**M1** **A1**	**2.4** **2.1**	1 partition with 1 subset and move E or write down {ABCD | E} 7 partitions with 2 subsets from part (a) Or listing these 7 partitions {ABC | DE} {ABD | CE} {ACD | BE} {AB | CDE} {AC | BDE} {AD | BCE} {A | BCDE}
			Alternative method 1 {ABCD | E} {ABC | DE} {ABD | CE} {ACD | BE} {AB | CDE} {AC | BDE} {AD | BCE} {A | BCDE}	 **M1** **A1**		 Any one correct partition Listing all 8 partitions with no repeats and no others
			Alternative method 2 $^4C_4 = 1$ = {ABCD} hence {ABCD | E} $^4C_3 = 4$ e.g. {ABC | D} hence {ABC | DE} e.g. {A | BCD} hence {A | BCDE} $^4C_2 \div 2 = 3$ e.g. {AB | CD} hence {AB | CDE}	 **M1** **A1**		 Showing how E can be added to {ABCD} to give {ABCD | E} Explaining why partitions of {ABCD} into 2 subsets generate 7 subsets of the required form
			Alternative method 3 A is in one subset and E in the other. For each of B, C and D there are 2 choices of subset $2^3 = 8$	 **M1** **A1**		 Identifying where 2 and 3 come from 2^3 or equivalent explanation of given result
			Alternative method 4 Subsets of sizes 1 and 4 $\Rightarrow$ either A or E is on its own (and everything else together) = 2 ways Subsets od sizes 2 and 3 $\Rightarrow$ smaller subset is one of A, E with one of B, C, D = 2×3 = 6 ways	 **M1** **A1**		 Explaining, without necessarily writing out the partitions, why there are 2 of this type (or 6 of the other type) Explaining both types Or variations on this using 1, 3, 3, 1 of the different types
				[2]		
7	(c)	(i)	{ABCDE} gives {ABCD | E} only	**B1** **[1]**	**1.1**	Identify {ABCDE} and show that it gives one new partition only
7	(c)	(ii)	e.g. {ABE | C | D} gives {AB | C | D | E}, {AB | CE | D} and {AB | C | DE}	**B1** **[1]**	**2.3**	Showing that a partition into more than 2 subsets will give more than 2 new partitions

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